Développements limités usuels

(au voisinage de 0)

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + o\left(x^n\right)$$

$$\begin{aligned}
\operatorname{ch} x &= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + o\left(x^{2n+1}\right) \\
\operatorname{sh} x &= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o\left(x^{2n+2}\right) \\
\operatorname{th} x &= x - \frac{x^3}{3} + \frac{2}{15} \cdot x^5 - \frac{17}{315} \cdot x^7 + o\left(x^8\right)
\end{aligned}$$

$$\begin{aligned}
\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n \cdot \frac{x^{2n}}{(2n)!} + o\left(x^{2n+1}\right) \\
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!} + o\left(x^{2n+2}\right) \\
\tan x &= x + \frac{x^3}{3} + \frac{2}{15} \cdot x^5 + \frac{17}{315} \cdot x^7 + o\left(x^8\right)
\end{aligned}$$

$$\begin{aligned}
(1+x)^\alpha &= 1 + \alpha . x + \frac{\alpha(\alpha-1)}{2} \cdot x^2 + \cdots + \frac{\alpha(\alpha-1)\ldots(\alpha-n+1)}{n!} \cdot x^n + o\left(x^n\right) \\
\frac{1}{1+x} &= 1 - x + x^2 + \cdots + (-1)^n x^n + o\left(x^n\right) \\
\frac{1}{1-x} &= 1 + x + x^2 + \cdots + x^n + o\left(x^n\right) \\
\frac{1}{\sqrt{1-x}} &= 1 + \frac{1}{2} \cdot x + \frac{3}{8} \cdot x^2 + \cdots + \frac{1.3.5\ldots(2n-1)}{2^n . n!} \cdot x^n + o\left(x^n\right)
\end{aligned}$$

$$\begin{aligned}
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n-1} \cdot \frac{x^n}{n} + o\left(x^n\right) \\
\ln(1-x) &= -x - \frac{x^2}{2} - \cdots - \frac{x^n}{n} + o\left(x^n\right) \\
\operatorname{argth} x &= x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2n+1}}{2n+1} + o\left(x^{2n+2}\right) \\
\arctan x &= x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + (-1)^n \cdot \frac{x^{2n+1}}{2n+1} + o\left(x^{2n+2}\right) \\
\operatorname{argsh} x &= x - \frac{1}{2} \cdot \frac{x^3}{3} + \frac{3}{8} \cdot \frac{x^5}{5} + \cdots + (-1)^n \cdot \frac{1.3.5\ldots(2n-1)}{2^n . n!} \cdot \frac{x^{2n+1}}{2n+1} + o\left(x^{2n+2}\right) \\
\arcsin x &= x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{3}{8} \cdot \frac{x^5}{5} + \cdots + \frac{1.3.5\ldots(2n-1)}{2^n . n!} \cdot \frac{x^{2n+1}}{2n+1} + o\left(x^{2n+2}\right)
\end{aligned}$$

NB : penser à écrire, notamment pour profiter de la formule de Stirling :

$$\frac{1.3.5\ldots(2n-1)}{2^n . n!} = \frac{1.2.3.4.5\ldots(2n-1).(2n)}{2.4\ldots(2n).2^n . n!} = \frac{(2n)!}{2^{2n}(n!)^2} = \frac{1}{2^{2n}}\binom{2n}{n} \underset{n\to\infty}{\sim} \frac{1}{\sqrt{\pi n}}$$